國立清華大學數學系訪問學者學術演講

NTHU Department of Mathematics Visiting Scholar Colloquium

講題　Estimates of Elliptic and Sub-elliptic Problems in Hardy Spaces

講者　張德健教授 (Georgetown University)

時間　2024.2.23 (Fri.) 11:00 – 12:00

地點　Room 201, General Building III

茶會　10:30, Room 707, General Building III

Abstract

The theory of singular integrals (SIO), introduced by Calderón and Zygmund as part of the theory of elliptic PDE's, has seen many extensions to different settings. Remaining within $\mathbb{R}^n$ as the ambient space, the variations introduced involve the following aspects, possibly also combined together:

(a) replace the standard dilations, $i.e.$, scalar multiplications, with non-isotropic ones;

(b) distinguish between a "$global$" theory and a "$local$" one;

(c) allow multi-parameter dilations.

The basic property that is common to all these types of singular integral operators is L^p-boundedness for $1 < p < \infty$ and $failure$ of L^p-boundedness, in general, for other values of p.

Hardy spaces H^p enter into this picture as the natural substitutes of L^p with $0 < p \leq 1$, allowing positive results about $H^p \to H^p$ and $H^p \to L^p$ boundedness of singular integrals for these values of p. The point is that each of the classes of SIO mentioned above admits its own Hardy spaces, so that, whenever a new class of SIO is introduced, it is natural to ask what are its Hardy spaces.

The second part of this talk, we with discuss an example which comes up naturally in the theory of elliptic boundary value problems. More precisely, let Ω be a bound domain in $\mathbb{R}^n$ with smooth boundary. Consider the following elliptic boundary valued problem:

$$\begin{cases} \Delta u = f \ \text{ in } \ \Omega \\ Xu = g \ \text{ on } \ \partial\Omega \end{cases}$$

Here X is a transversal vector field to the boundary. This includes the regular Dirichlet and Neumann problem. We introduce suitable Hardy spaces $H^p(\Omega)$ and BMO space $BMO(\Omega)$ on a "suitable" domain in $\mathbb{R}^n$. Then we shall show that

$$\left\| \frac{\partial u}{\partial x_j \partial x_k} \right\|_{H^p(\Omega)} \leq C_p \|f\|_{H^p(\Omega)}$$

for $0 < p < \infty$.

歡迎參加，敬請張貼

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